Application of Simulated Annealing Method on Tabarru-Fund Valuation using Inflator by Vasicek Model Approach Based on Profit and Loss Sharing Scheme

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Abstract
Currently, the financial services industry is dominated by conventional banks and individuals that apply the system of interest or an excess of loans. In Islam, this excess is referred to as usury, which is prohibited by Islamic law because, in practice, usury makes borrowers poorer as they cannot pay such high-interest installments. Not to mention, late payments are subject to penalties that will continue to accumulate if the borrower is unable to pay the next installment. From these facts, this system is prohibited by Islamic Law because there are harmed parties. Therefore, this research discusses mathematical models in the form of Islamic investment business loans for micro-economic traders by implementing a profit and loss sharing system.

Tabarru-fund is a set of funds derived from borrowers’ contributions used to overcome conditions when they experience losses in certain conditions. In this mathematical model, the tabarru-fund acts as the premium that must be paid if the borrower is still profitable after the principal installments have paid off. This sharia model with tabarru funds is obtained by calculating the premium which involves the problem of minimizing the remaining tabarru funds in a certain period. The future value of the trader's profit rate will be projected using the Vasicek Model approach which previously determined the parameter estimation using OLS regression and then the data is generated using Monte Carlo simulation so that the sharia inflator is obtained. This sharia inflator plays a role in the optimization process of minimizing the remaining tabarru-fund which will be solved by the Simulated Annealing (SA) algorithm.

Keywords: investment, sharia, profit sharing, tabarru, inflator, Vasicek Model, OLS, Monte Carlo, Simulated Annealing

1. Introduction
The development of Islamic banking financial institutions in Indonesia began in the 1990s and experienced increasingly widespread development in the early 2000s [1]. Marked by the emergence of several Islamic banks established by conventional banks, both publicly and privately owned. Positive and stable economic performance has provided a large space for the development of Islamic economics in Indonesia. The presence of Islamic economics in Indonesia, where the majority of the population is Muslim, is a new point in the history of the national economy. Economic activity in the financial services industry, especially in the banking sector, both conventional and individual, can never be separated from the practice of using interest returns or what is often referred to as usury. Loans with interest systems sometimes do not make the economy grow and develop better. For example, in the micro economy, small sellers in traditional markets often make loans to moneylenders who apply a
high-interest system. Not to mention, sometimes moneylenders as lenders apply a penalty system if the borrower is unable to pay the loan at the due date. If this is the case, instead of being able to develop his business, the borrower will experience even greater losses due to swollen debts, which make the borrower poorer. Back to the concept of Islamic law, Islam clearly prohibits usury (interest), gharar (uncertainty), qimar (gambling), and myisur (fraud) in the implementation of economic activities [1]. So, Islam offers a concept of lending and borrowing money without interest, and replaces it with a gift that is given to the lender as an appreciation for the investment he provides. Therefore, because profit sharing is in the form of a gift, the amount is not fixed, depending on the condition of the borrower at that time.

In this research, the author intends to develop a sharia-based mathematical model based on previous research [2-3] with the addition of tabarru-fund. The effort to establish the Islamic banking system is based on the prohibition in Islam to collect or borrow with interest or what is called usury and the prohibition to invest in businesses that are categorized as haram (e.g. gambling), which is cannot be guaranteed in the conventional banking system [4]. Currently, the number of micro-economic actors occupies a larger portion compared to macro and medium-economic actors. The micro economy occupies the largest number of nearly 42 million out of a total of 43 million in Indonesia's total economic structure. Most of the difficulties faced by micro-economic actors are access to capital as much as approximately 40% of the total difficulties faced. Through the support of access to capital, micro-businesses can increase their income by an average of 87% per month [5]. According to [6], at least three principles in the operation of Islamic banks are different from conventional banks, especially in service to customers, which must be maintained by bankers, namely the principle of justice, the principle of equality, and the principle of tranquility.

Moneylenders act as non-bank financial institutions that are engaged in financial services, self-employed, but not incorporated, and manage their businesses, with their policies and regulations [7]. Moneylenders can provide easy, fast, and no collateral (based on the principle of mutual trust) loans. However, the set interest rates are usually too high at around 10-30% per two months [2] and the collection of loans is sometimes done with arbitrary actions if the borrower cannot pay the installments. In 2014, a study addressed the fundamental question that if profit-and-loss sharing (PLS) is applied to the banking system, it can be more welfare-enhancing than interest, by developing rigorous modeling [8]. The study measured the effect of income distribution on borrowers and lenders. Under certain production conditions and competitive markets, the results suggest that both PLS and interest systems are equally efficient. However, when the situation is uncertain such as production shocks, only the PLS system can overcome these conditions, because the PLS system distributes risk fairly at the individual level between borrowers and lenders. Research [9] evaluates the failure of PLS-contracts in Islamic banking and its potential to be developed within the New Institutional Economic Theory (NIE) scope. In [10] with the case of an auction, the proportion of profit-and-loss sharing in Profit Sharing Contract (PSC) was studied. The results show that the expected returns of sellers with nontrivial profit sharing are greater than those of auctions with only one-time payments.

In 2014, an Islamic model with profit sharing was developed by determining the objective function to maximize the investor's return and maximize the trader's share [2-3]. In addition, the optimal values for loan size and the number of loan periods were determined. In [11], the Vasicek Model equation was discretized, which can be used to describe dynamic interest rates. This model is included in the stochastic differential equation that can describe fluctuations in interest rate movements. In addition, this model can also be used to see the magnitude of interest rates in the future period. Therefore, this research intends to calculate the value of a tabarru-fund by involving the Vasicek Model in the calculation process.
2. Vasicek model

The Vasicek model was first introduced by Oldrich Vasicek in 1977. This model is included in the stochastic differential equation that can describe fluctuations in interest rate movements. In addition, this model can also be used to see the magnitude of interest rates in the future period.

Vasicek’s classic dynamic interest rate model r(t) is [12]:

$$dr(t) = \alpha (\beta - r(t))dt + \sigma dW(t)$$  \hspace{1cm} (1)

with

- \(\alpha\) = speed of interest rate towards level \(\beta\)
- \(\beta\) = long run interest rate
- \(r(t)\) = interest rate
- \(\sigma\) = interest rate volatility
- \(W(t)\) = standard Brownian motion (Wiener process)

2.1. Vasicek model solutions

To determine the explicit formula of \(r(t)\), first define the process \(X(t)\) where \(X(t) = r(t) - \beta\).

By substituting \(r(t) = X(t) + \beta\) into Eq. (1), the solution of the stochastic differential equation is obtained as follows:

$$d(\ X(t) + \beta) = -\alpha (X(t) + \beta)dt + \sigma dW(t)$$

$$dX(t) = -\alpha X(t)dt + \sigma dW(t)$$ \hspace{1cm} (2)

Where \(X(t)\) is an Ornstein-Un-Debeck process which has the following solution:

$$X(t) = X(0)e^{-\alpha t} + \sigma e^{-\alpha t} \int_0^t e^{\alpha s} dW(s)$$ \hspace{1cm} (3)

From the Ornstein-Un-Debeck process, as \(X(t) = r(t) - \beta\), then the solution of the Vasicek model is obtained as follows

$$r(t) = r(0)e^{-\alpha t} + \beta (1 - e^{-\alpha t}) + \sigma e^{-\alpha t} \int_0^t e^{\alpha s} dW(s)$$ \hspace{1cm} (4)

2.2. Discretization of the Vasicek model stochastic differential equation

Note that \(W(t)\) is a Brownian motion, so we can define \(dW(t) = Z\sqrt{dt}, Z \sim N(0,1)\) \cite{13}. The discretization of equation (4) is found to be

$$r(t) = r(0)e^{-\alpha t} + \beta (1 - e^{-\alpha t}) + \sigma \sqrt{\frac{1}{2\alpha}(1-e^{-2\alpha t})}Z$$

with \(r(0)\) is the short rate when \(t = 0\), then we get

$$r(t) = r(u)e^{-\alpha(t-u)} + \beta (1 - e^{-\alpha(t-u)}) + \sigma \sqrt{\frac{1}{2\alpha}(1-e^{-2\alpha(t-u)})}Z.$$  \hspace{1cm} (5)
Where Eq. (5) \( \forall u \leq t, r(t) \) is normally distributed with mean \( r(u)e^{-\alpha(t-u)} + \beta(1 - e^{-\alpha(t-u)}) \) and variance \( \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha(t-u)}) \).

In order to simulate \( r \) when \( t_1, t_2, \cdots, t_n \) using Euler approach, we define

\[
\begin{align*}
\frac{dr(t_{i+1})}{dt} &= r(t_i)e^{-\alpha(t_{i+1}-t_i)} + \beta(1 - e^{-\alpha(t_{i+1}-t_i)}) + \sigma\sqrt{\frac{1}{2\alpha}(1 - e^{-2\alpha(t_{i+1}-t_i)})Z_{i+1}} \\
&= r(t_i)e^{-\alpha t} + \beta(1 - e^{-\alpha t}) + \sigma\sqrt{\frac{1}{2\alpha}(1 - e^{-2\alpha t})Z_{i+1}}
\end{align*}
\]

where \( Z_{i+1} \sim N(0,1) \) and \( i = 0,1,\ldots,n-1 \) [11]. For a constant time interval difference, in this case, when \( \Delta t = t_{i+1} - t_i = 1 \), then

\[
\begin{align*}
r(t_{i+1}) &= r(t_i)e^{-\alpha} + \beta(1 - e^{-\alpha}) + \sigma\sqrt{\frac{1}{2\alpha}(1 - e^{-2\alpha})Z_{i+1}}
\end{align*}
\]

Furthermore, the alpha, beta, and sigma parameter estimation will be determined. The equation above is a simple linear regression which can be solved using Ordinary Least Square (OLS) estimation [14] by assuming

\[
\begin{align*}
\gamma_0 &= \beta(1 - e^{-\alpha}), \quad \gamma_1 = e^{-\alpha}, \quad \text{and } u_{i+1} = \sigma\sqrt{\frac{1}{2\alpha}(1 - e^{-2\alpha})Z_{i+1}}.
\end{align*}
\]

Thus,

\[
\begin{align*}
r(t_{i+1}) &= \gamma_1 r(t_i) + \gamma_0 + u_{i+1} = \gamma_0 + \gamma_1 r(t_i) + u_{i+1}
\end{align*}
\]

So we have,

\[
\begin{align*}
\hat{\gamma}_1 &= \frac{n\sum r(t_i)r(t_{i+1}) - \sum r(t_i)\sum r(t_{i+1})}{n\sum (r(t_i))^2 - (\sum r(t_i))^2} \quad \text{and} \quad \hat{\gamma}_0 = \frac{\sum (r(t_i))^2\sum r(t_{i+1}) - \sum r(t_i)\sum r(t_{i+1})}{n\sum (r(t_i))^2 - (\sum r(t_i))^2}
\end{align*}
\]

Then

\[
\hat{\alpha} = \ln \frac{1}{\hat{\gamma}_1} \quad \text{and} \quad \hat{\beta} = \frac{\hat{\gamma}_0}{1 - \hat{\gamma}_1}
\]

Remember that \( \text{Var}(r(t)) = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha t}) \), then by assuming that the amount of observation is big enough, we get

\[
\hat{\sigma} = \sqrt{2\hat{\alpha}\text{Var}(r(t))}
\]

3. Sharia inflator construction

The sharia inflator can be constructed by projecting the equivalent rate value from available profits.
3.1. Equivalent rate

In the Islamic financial system, there is no interest system (riba). In this case, the profit earned is shared according to the agreement of both parties (lender and borrower) which is hereinafter known as the profit sharing ratio. The percentage of profit shared with the owner of the fund is expressed in the equivalent rate, which is then written as $r(t)$, which states the value of the equivalent rate in period $t$. The Vasicek model is used to construct the sharia inflator.

Suppose the fund owner provides investment capital to a trader in the amount of $x_0$, which can be seen in the following table:

**Table 1. Funds.**

<table>
<thead>
<tr>
<th>Period</th>
<th>Amount of funds</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$x_0$</td>
<td>Initial capital is given to traders.</td>
</tr>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>The profit earned at the end of the first period: $y_1 = x_1 - x_0$</td>
</tr>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>$y_2 = x_2 - x_1$</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>$y_3 = x_3 - x_2$</td>
</tr>
<tr>
<td>⋮</td>
<td>⋮</td>
<td>⋮</td>
</tr>
<tr>
<td>$t$</td>
<td>$x_t$</td>
<td>$y_t = x_t - x_{t-1}, \forall t = 1,2,...$</td>
</tr>
</tbody>
</table>

It is known that in a business activity, the amount of profit is uncertain so it can be assumed that the profit is stochastic.

3.2. Inflator equivalent rate ($r(t)$)

In this research, we assume that the equivalent rate follows the discretized Vasicek model equation with interval difference $t_{i+1} - t_i = 1$, which is

$$r(t_{i+1}) = r(t_i) e^{-\alpha} + \beta (1 - e^{-\alpha}) + \sigma \sqrt{\frac{1}{2\alpha}} (1 - e^{-2\alpha}) Z_{i+1}$$

Substitute the Eq. (6) into the stochastic sharia inflator model

$$\varphi = \begin{bmatrix} 1 \\ 1 + r(1) \\ (1 + r(1))(1 + r(2)) \\ \vdots \\ \prod_{k=1}^{t} (1 + r(k)) \end{bmatrix} = \begin{bmatrix} \varphi_0 \\ \varphi_1 = \varphi_0 (1 + r(1)) \\ \varphi_2 = \varphi_1 (1 + r(2)) \\ \vdots \\ \varphi_t = \varphi_{t-1} (1 + r(k)) \end{bmatrix}$$

Thus,

$$\varphi_t = \begin{cases} 1 & ; t = 0 \\ 1 + r(1) & ; t = 1 \\ \prod_{k=2}^{t} \left( 1 + r(k) e^{-\alpha} + \beta (1 - e^{-\alpha}) + \sigma \sqrt{\frac{1}{2\alpha}} (1 - e^{-2\alpha}) Z_k \right) & ; t > 1 \end{cases}$$
4. Optimization of tabarru-fund using Simulated Annealing algorithm

In this part, we will discuss the objective function that will be used in the optimization of minimizing tabarru-fund.

4.1. Simulated Annealing algorithm

This research is a development from the previous research [2-3], so the optimization method used is the same. The Simulated Annealing (SA) method is a minimization method commonly used to find the global minimum price of a function, which is the lowest minimum price of a function. The SA method is one of the neighborhood search methods that allow inferior solutions to be accepted. The benefits of Simulated Annealing are its easy implementation and its possibility of finding a global optimal even after finding a local minimum, as it accepts solutions that are worse than the best candidate [15].

SA is also one type of meta-heuristic algorithm, which is a high-level procedure or heuristic that is not designed based on complex theory, and this method also does not guarantee that we get a really good (optimal) solution. However, this heuristic algorithm can give a very good feasible solution quickly and arrive at a point that is very close to the solution [16].

<table>
<thead>
<tr>
<th>(1) Set initial temperature $T = T_0$ and initial variable $x_0$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) Consider the initial variable $x_0$ as the current variable $x_i$.</td>
</tr>
<tr>
<td>(3) Calculate the current function $f(x_i)$.</td>
</tr>
<tr>
<td>(4) Randomly generate new variable $x_{i+1}$.</td>
</tr>
<tr>
<td>(5) Calculate the new function $f = f(x_{i+1})$.</td>
</tr>
<tr>
<td>(6) Generate a random number $p$ on the interval $[0,1]$.</td>
</tr>
<tr>
<td>(7) while $T &gt; 1$</td>
</tr>
<tr>
<td>$x_{i+1} = x_i + \text{randn}();$</td>
</tr>
<tr>
<td>$f(x_{i+1}) = fobj;$</td>
</tr>
<tr>
<td>$p = \text{rand}();$</td>
</tr>
<tr>
<td>if $p &lt; \exp\left(\frac{f(x_i) - f(x_{i+1})}{T}\right)$</td>
</tr>
<tr>
<td>$f(x_i) = f(x_{i+1});$</td>
</tr>
<tr>
<td>$x_i = x_{i+1};$</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>$T = T \times (1 - \text{coolingrate})$</td>
</tr>
<tr>
<td>end</td>
</tr>
</tbody>
</table>

**Figure 1.** Simulated Annealing algorithm.

4.1.1. Annealing schedule

There are four components of the annealing schedule or cooling schedule, namely the initial temperature, final temperature, temperature reduction rules, and iterations at each certain temperature [17].

**Initial temperature**

The initial temperature must be high enough to allow transfer to neighboring states. If the selected temperature is not high enough, then the solution that will be obtained will only be the same (or very close) to the initial solution. However, choosing too high temperature can cause
the search to move around to almost all neighbors and transform the search into a random search. So, the technique used is usually a heuristic. If the maximum difference (difference from the objective function) is known between one neighbor and another, then this information can be used as initial information in calculating the initial temperature.

**Final temperature**

Ideally, the optimization process stops when the system is at the freezing point, which is a state when there is no change in the system's energy.

**Temperature reduction rules**

The theory explains that at any given temperature, sufficient iterations are needed so that the system is in a stable state at that temperature [18]. To get the global minimum can be done either by doing a lot of iterations at a small number of temperatures or doing a few iterations but a large number of temperatures, or both are balanced.

**Iterations at each certain temperature**

Research [19] suggests only doing one iteration at each temperature, but the temperature drop is carried out very slowly. Another alternative, by dynamically changing the number of iterations at each temperature during the algorithm. At low temperatures, it is important to apply a large number of iteration counts, so that the local optimum can be fully extracted. Whereas at high temperatures, the number of iterations can be smaller.

4.1.2. Comparison with other methods

Any efficient optimization algorithm must use two techniques to find a global maximum: exploration to investigate new and unknown areas in the search space, and exploitation to make use of knowledge found at points previously visited to help find better points. These two requirements are contradictory, and a good search algorithm must find a tradeoff between the two. Herewith, the comparison of Simulated Annealing with other method [20].

**Neural nets**

The main difference compared with neural nets is that neural nets learn (how to approximate a function) while simulated annealing searches for a global optimum. Neural nets are flexible function approximators while SA is an intelligent random search method. The adaptive characteristics of neural nets are a huge advantage in modelling changing environments. However, the power-hungriness of SA limits its use as a real-time application.

**Genetic algorithms**

Direct comparisons have been made between ASA/VFSR and publicly-available genetic algorithm (GA) codes, using a test suite already adapted and adopted for GA. In each case, ASA outperformed the GA problem. GA is a class of algorithms that are interesting in their own right; GA was not originally developed as an optimization algorithm, and basic GA does not offer any statistical guarantee of global convergence to an optimal point. Nevertheless, it should be expected that GA may be better suited for some problems than SA.

**Random search**

The brute force approach for difficult functions is a random, or an enumerated search. Points in the search space are selected randomly, or in some systematic way, and their fitness evaluated. This is an unintelligent strategy, and is rarely used by itself.

**Gradient methods**

A number of different methods for optimizing well-behaved continuous functions have been developed which rely on using information about the gradient of the function to guide the direction of search. If the derivative of the function cannot be computed, because it is discontinuous, for example, these methods often fail. Such methods are generally referred to as
hillclimbing. They can perform well on functions with only one peak (unimodal functions). But on functions with many peaks, (multimodal functions), they suffer from the problem that the first peak found will be climbed, and this may not be the highest peak. Having reached the top of a local maximum, no further progress can be made.

**Iterated search**

Random search and gradient search may be combined to give an iterated hillclimbing search. Once one peak has been located, the hillclimb is started again, but with another, randomly chosen, starting point. This technique has the advantage of simplicity and can perform well if the function does not have too many local maxima. However, since each random trial is carried out in isolation, no overall picture of the "shape" of the domain is obtained. As the random search progresses, it continues to allocate its trials evenly over the search space. This means that it will still evaluate just as many points in regions found to be of low fitness as in regions found to be of high fitness.

Both SA and GAs, by comparison, start with an initial random population, and allocate increasing trials to regions of the search space found to have high fitness. This is a disadvantage if the maximum is in a small region, surrounded on all sides by regions of low fitness. This kind of function is difficult to optimize by any method, and here the simplicity of the iterated search usually wins.

### 4.2. Tabarru-fund optimization

Given a Profit Loss Sharing model with the addition of tabarru-fund, which satisfies the following conditions [2]:

1. In the condition that the borrower still has profit after paying off the principal installment, the borrower pays the amount $S(t)$ plus the premium payment (in this case is the tabarru fund).
2. In the condition of a loss, the borrower does not have to pay anything, but $S(t) = I_p$ is taken from the tabarru-fund that has been collected.

Thus, it is hoped that the borrower will never have an installment debt or $H(t) = 0$ for all $t$.

Referring to [3], the characteristics of this model are the same as the PLS model in the previous section, but there are some differences related to the addition of tabarru-fund, as follows:

1. This model completes the realization of profit sharing and loss sharing explicitly.
2. The amount of the tabarru-fund that is paid as a premium on the condition if the borrower earns a net profit that is greater than the principal installment plus profit sharing, i.e. $w(t) > I(t) + B(t)$. If the opposite happens, then the inability to pay the principal installment will be paid by taking from the collected tabarru-fund.
3. There is no debt at any time $t$, $t=1,2,...,T$ because when there is an inability to pay the principal installment, it has been fulfilled by the tabarru-fund.

In this model, there are additional variables, namely benefits (compensation) and premiums (tabarru-fund), which result in no debt during period $t$. Profit sharing on day $t$, $B(t)$ is paid if the day's principal installment $I(t)$ is fulfilled, with the determined amount of portion $p$. The benefit, $b(t)$ is given to traders who are unable to pay the principal installment due to insufficient profit. In this model, traders are required to pay tabarru-funds, $Q(t)$ if the net profit condition is greater than the amount of principal installments, profit sharing, and estimated tabarru-fund.
The formulation of the amount of compensation, profit sharing, and tabarru-fund is given as follows:

\[
\begin{align*}
    b(t) &= \begin{cases} 
        0 & ; w(t) > I_p \\
        I_p - w(t) & ; 0 < w(t) \leq I_p \\
        I_p & ; w(t) \leq 0 
    \end{cases} \\
    B(t) &= \begin{cases} 
        p(w(t) - I(t)) & ; w(t) - I(t) > 0 \\
        0 & ; w(t) - I(t) \leq 0 
    \end{cases} \\
    Q(t) &= \begin{cases} 
        aPr & ; w(t) > I(t) + B(t) \\
        0 & ; w(t) \leq I(t) + B(t) 
    \end{cases}
\]

The amount of the tabarru-fund are set so that the difference between the tabarru-fund and the sum of benefits is zero, which means that the future value of the remaining tabarru-fund is zero.

\[
S(t) = I(t) + B(t) + Q(t)
\]

\[
FVsdt = \sum_{i=1}^{T} (Q(i) - b(i))(1 + r_{ai})^{T-1} \approx 0
\]

Then,

\[
FVsdt = \sum_{i=1}^{T} (Q(i) - b(i)) \varphi_i \approx 0.
\]

The problem in this model is to determine the amount of premium that will cause \( FVsdt \) to be at or near zero. The average premium will be determined from historical profit data. The average premium multiplier \( a \) is searched to obtain the optimal premium amount which aims to minimize the remaining tabarru-fund. Furthermore, the problem of minimizing the remaining tabarru-fund will be solved by the Simulated Annealing (SA) algorithm with an objective function:

\[
\min F(a) = -\frac{1}{1 + sd(a)}
\]

constraints: \( sd(a)=0 \) and \( a > 0 \) where \( a \in \mathbb{R} \).

The optimization problem in Eq. (11) is converted into an unconstrained optimization problem by applying a penalty function, so the objective function becomes:

\[
\min F(a) = -\frac{1}{1 + sd(a)} + \mu_1 H_1[sd(a)](sd(a))^2 + \mu_2 H_2[a](a)^2
\]

with

\[
H_1[sd(a)] = \begin{cases} 
1 & , sd(a) \neq 0 \\
0 & , sd(a) = 0 
\end{cases}
\]

\[
H_2[a] = \begin{cases} 
1 & , a \leq 0 \\
0 & , a > 0 
\end{cases}
\]

\( \mu_1, \mu_2 \) large positive number

\( sd(a) = \sum_{i=1}^{T} (aPr - b(i)) \varphi(i) \)

and Pr is the paid average premium.

In this case, if the constraint is not fulfilled, or \( a < 0 \), the value of \( F(a) \) will be so large that it will not be selected as the minimum value.
5. Model simulation
Simulations will be carried out using secondary data and with the help of the MATLAB program.

5.1. Data description
The data used is secondary data from previous research [2] which contains the daily profit data of traders for 60 days. The traders are given an investment (A) in the form of a capital loan of IDR 1,000,000 by an Islamic financial institution which will be returned in installments for 60 days (T) with a profit sharing ratio of 35:65 (35% share for investors and 65% share for traders), so that the amount of fixed installments is $I_p \approx 16,666.67$. Furthermore, considered the trader's daily profit data as follows.

<table>
<thead>
<tr>
<th>day</th>
<th>profit $w(t)$</th>
<th>compensation $b(t)$</th>
<th>equivalent rate $r(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>885,000</td>
<td>0</td>
<td>0.1363</td>
</tr>
<tr>
<td>2</td>
<td>880,000</td>
<td>0</td>
<td>0.1355</td>
</tr>
<tr>
<td>3</td>
<td>-845,000</td>
<td>16,666,67</td>
<td>-0.1301</td>
</tr>
<tr>
<td>4</td>
<td>860,000</td>
<td>0</td>
<td>0.1324</td>
</tr>
<tr>
<td>5</td>
<td>915,000</td>
<td>0</td>
<td>0.1409</td>
</tr>
<tr>
<td>6</td>
<td>-600,000</td>
<td>16,666,67</td>
<td>-0.0924</td>
</tr>
<tr>
<td>7</td>
<td>-225,000</td>
<td>16,666,67</td>
<td>-0.0347</td>
</tr>
<tr>
<td>8</td>
<td>140,000</td>
<td>0</td>
<td>0.0216</td>
</tr>
<tr>
<td>9</td>
<td>100,000</td>
<td>0</td>
<td>0.0262</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>59</td>
<td>-580,000</td>
<td>16,666,67</td>
<td>-0.0893</td>
</tr>
<tr>
<td>60</td>
<td>540,000</td>
<td>0</td>
<td>0.0832</td>
</tr>
</tbody>
</table>

From the data above, it is obtained that the amount of tabarru-fund is around IDR 12,598.04.

5.2. Sharia inflator projection of the equivalent rate

5.2.1. Rate projection
After performing simple calculations as in Table 2, the process will continue by projecting the value of the future period rate using the Vasicek Model, as below.

$$ r(t_{i+1}) = 0.1671940776r(t_i) + 0.0229021629 + 0.104933849Z_{t_{i+1}} \quad (13) $$

5.2.2. Sharia inflator modeling
From the rate projection in Eq. (13), the sharia inflator model based on the Vasicek Model can be determined as follows:

$$ \phi_i = \begin{cases} 
1 & : t = 0 \\
1 + r(1) & : t = 1 \\
\prod_{k=2}^{t} (1 + 0.1671940776r(t_{k-1}) + 0.0229021629 + 0.104933849Z_k) & : t > 1 
\end{cases} $$
5.2.3. Projection of future sharia inflator

Table 3 shows the projection of the sharia inflator from the trader's daily profit data for 60 days with 1000 repetitions.

Table 3. Projection data for the next 60 days.

<table>
<thead>
<tr>
<th>day</th>
<th>( r(t) ) projection</th>
<th>sharia inflator</th>
<th>( b(t) ) projection</th>
<th>compensation projection</th>
<th>( B(t) ) projection</th>
<th>( Q(t) ) estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0586</td>
<td>10,586</td>
<td>380,519.48</td>
<td>0</td>
<td>4,816.44</td>
<td>12,598.04</td>
</tr>
<tr>
<td>2</td>
<td>0.0536</td>
<td>11,153</td>
<td>348,051.95</td>
<td>0</td>
<td>4,405.48</td>
<td>12,598.04</td>
</tr>
<tr>
<td>3</td>
<td>0.0485</td>
<td>11,694</td>
<td>314,935.06</td>
<td>0</td>
<td>3,986.30</td>
<td>12,598.04</td>
</tr>
<tr>
<td>4</td>
<td>0.0485</td>
<td>12,262</td>
<td>314,935.06</td>
<td>0</td>
<td>3,986.30</td>
<td>12,598.04</td>
</tr>
<tr>
<td>5</td>
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<td>12,791</td>
<td>280,519.48</td>
<td>0</td>
<td>3,550.68</td>
<td>12,598.04</td>
</tr>
<tr>
<td>6</td>
<td>0.0366</td>
<td>13,259</td>
<td>237,662.34</td>
<td>0</td>
<td>3,008.22</td>
<td>12,598.04</td>
</tr>
<tr>
<td>7</td>
<td>0.0352</td>
<td>13,726</td>
<td>228,571.43</td>
<td>0</td>
<td>2,893.15</td>
<td>12,598.04</td>
</tr>
<tr>
<td>8</td>
<td>0.0263</td>
<td>14,087</td>
<td>170,779.22</td>
<td>0</td>
<td>2,161.64</td>
<td>12,598.04</td>
</tr>
<tr>
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<td>-0.0021</td>
<td>17,944</td>
<td>-13,636.36</td>
<td>16,666.67</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60</td>
<td>-0.0111</td>
<td>17,744</td>
<td>-72,077.92</td>
<td>16,666.67</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

5.3. Tabarru-fund valuation

Optimization will be carried out using the data in Table 3 to determine the amount of the average premium (tabarru-fund) multiplier, namely \( a \) by using the objective function in Eq. (12) and the Simulated Annealing method so that the following output is obtained.

\[
\begin{align*}
\text{>> sa} \\
\text{nilai a:} \\
0.70109395 \\
\text{nilai sdct(a):} \\
-27.88368702 \\
\text{Dana tabarru yang harus dibayarkan:} \\
8832.41 \\
\text{Besarnya sisa dana tabarru / fobj(a):} \\
0.03719728
\end{align*}
\]

Figure 2. MATLAB output of tabarru-fund valuation.

From the output in Figure 2, it can be concluded that the amount of tabarru-fund that must be paid in a loan period is IDR 8,832.41 with the remaining tabarru-fund is 0.03719728 \( \approx 0 \).

Furthermore, by using the condition in Eq. (9) the amount of profit sharing that will be received by the owner of capital can be obtained. While the estimated amount of the trader's daily installments for the next 60 days can be seen in Table 4.
6. Conclusion
The Vasicek model can be used to describe the movement of the equivalent rate in the future period. The estimation of parameter values in the Vasicek model can be determined using OLS regression, OLS can calculate parameter values with an error of 0.104933849. The sharia inflator is formed by utilizing the Vasicek Model and OLS regression where the data is generated by Monte Carlo simulation 1000 times to get the closest value.

The Simulated Annealing method is used to calculate the minimum value of a function and in this research, the function reaches a minimum when $a = 0.70109395$ with the minimum value is $0.03719728$. Based on the concept in Islamic finance that prohibits usury, tabarru-fund can be utilized to replace the premium position in conventional financial concepts.

Acknowledgments
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References


